Close out and Final report for
NASA Glenn Cooperative Agreement NCC3-453

Multidisciplinary Design Optimization for Aeropropulsion Engines and Solid Modeling/Animation via the Integrated Forced Methods

The grant closure report is organized in the following four chapters.

- Chapter describes the two research areas
- Design optimization and Solid mechanics
- Ten journal publications are listed in the second chapter
- Five highlights is the subject matter of chapter three

CHAPTER 1
Research Topic-1

Description: The Design Optimization Test Bed CometBoards

The research to compare different optimization algorithms and alternate analysis methods for structural design applications has grown into a multidisciplinary design test bed that is still referred to by its original acronym, CometBoards, which stands for COMparative Evaluation Test Bed of Optimization and Analysis Routines for the Design of Structures. The modular organization of CometBoards, shown in figure 1, allows innovative methods (or computer codes) to be tested quickly through its soft coupling feature. Optimizers and analyzers are two important modules of CometBoards. The optimizer module includes a number of algorithms:

- the fully utilized design;
- optimality criteria methods;
- the method of feasible directions;
- the modified method of feasible directions;
- three different sequential quadratic programming techniques;
- the Sequential Unconstrained Minimizations Technique;
- sequential linear programming;
- a reduced gradient method; and others.

Likewise, the analyzer module includes:

- COSMIC/NASTRAN;
- the nonlinear analyzer MHOST;
- the U.S. Air Force ANALYZE/DANALYZE code;
- IFM/ANALYZERS;
- the aircraft flight optimization analysis code FLOPS ;
- the NASA Engine Performance Program NEPP; and others.

Some of the other unique features of CometBoards are:

- a multiple optimizer cascade strategy;
- design variable and constraint formulations;
- a global scaling strategy;
- analysis and sensitivity approximations through regression and neural networks;
- substructure optimization on sequential as well as parallel computational platforms.

CometBoards has provisions to accommodate up to 10 different disciplines, each of which can have a maximum of 5 subproblems. The test bed can optimize a large system, which can be defined in as many as 50 different subproblems. Alternatively, a component of a large system can be optimized. The design test bed has been successfully used to solve a number of multidisciplinary problems. The CometBoards test bed has over 50 numerical examples. It is written in Fortran 77, except for the neural network code, Cometnet , which is written in the C++ language. The C++ code is integrated into the CometBoards Fortran code through soft-coupling. Soft-coupling is achieved by first generating an executable file from the Cometnet C++ source code; then Cometnet is invoked from CometBoards through a system call. Information is exchanged between the two programs through data files. CometBoards is available on UNIX-based SGI and Sun workstations. CometBoards is continuously being improved to increase its reliability and robustness for optimization at system as well as at component levels. Stochastic calculations are being implemented into CometBoards.

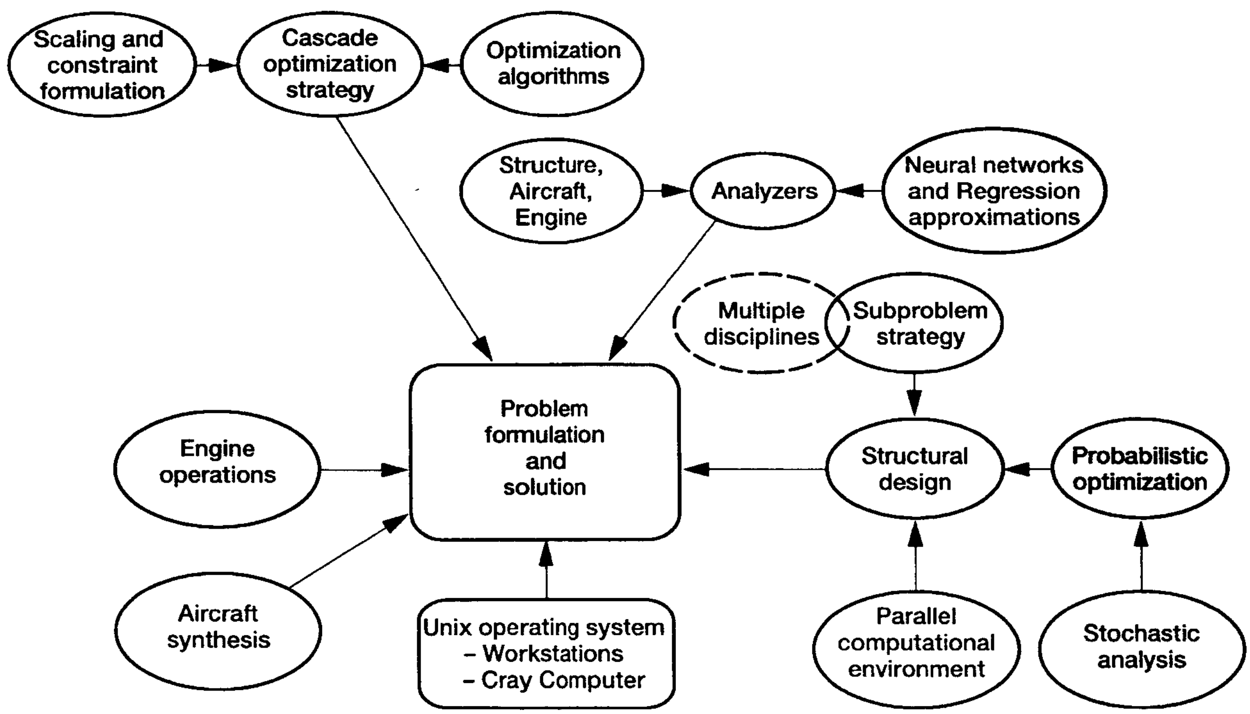
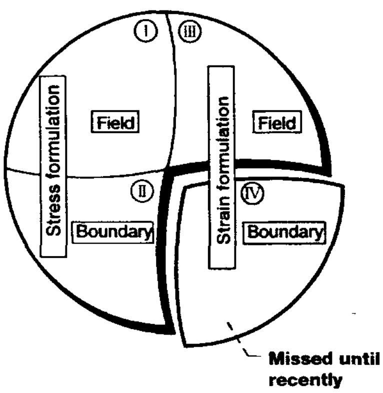


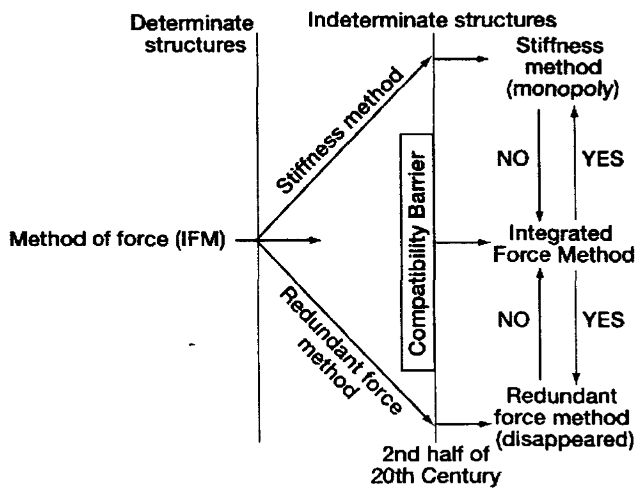
Figure 1. Organization of Design Optimization Test-bed CometBoards

CHAPTER 2
Research Topic-2

Solid Mechanics: Integrated Force Method of Analysis

The stress and the strain formulations; (and the material law) are fundamental solid mechanics concepts. In 1822, Cauchy formulated the equilibrium equations or the stress formulation in the field $(\tau ij, j + bi = 0)$ as well as on the boundary $(\tau ij, j + bi = 0)$ of an elastic continuum. In 1860, St. Venant provided the compatibility condition or the strain formulation, but only in the field of the continuum. Recently, the strain formulation on the boundary or the boundary compatibility conditions has been formulated by Patnaik. The pie chart depicts the two formulations. The discipline has acknowledged the existence of compatibility conditions, but it has never been adequately researched, neither understood nor used. The 'cut & close' redundant-force based compatibility conditions in discrete structural analysis has little semblance to the strain formulation in elasticity. The CC have been showcased. Used sparingly; confused with the continuity condition. Analysis progressed without a total comprehension of the compatibility conditions.





Stress and strain formulations **Compatibility restrained the growth of force method**

The equilibrium equations have been manipulated to obtain the Navier's formulation in elasticity and the stiffness method in finite element analysis. The displacement method was justified because the strain formulation in the field was automatically satisfied in displacement variables. The boundary compatibility conditions are nontrivial is displacements. $(\Re_1 \neq 0, \Re_2 \neq 0\Re_2 \neq 0)$.

Boundary compatibility conditions in elasticity when expressed in displacements (u, v, w) yield nontrivial conditions.

$$R_1 = a_{vy}\frac{\partial^2 v}{\partial z^2} + a_{vz}\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2}{\partial y \partial z}\left(a_{vz}v + a_{vy}w\right) = 0$$

$$R_2 = a_{vz}\frac{\partial^2 w}{\partial x^2} + a_{vx}\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2}{\partial z \partial x}\left(a_{vx}w + a_{vz}u\right) = 0$$

$$R_3 = a_{vy}\frac{\partial^2 v}{\partial x^2} + a_{vx}\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2}{\partial x \partial y}\left(a_{vy}u + a_{vx}v\right) = 0$$

The boundary conditions of the new force method (or the completed Beltrami Michell's formulation) expressed in displacement variables are not identical to the traditional conditions of the stiffness method. For example, the traditional clamped boundary condition for a rectangular plate in flexure is specified in terms of displacement and slope. However, in the force method these are specified in moments; which when expressed in displacement variable become conditions in curvatures.

Displacement method in continuum as well as finite element model, (especially at the flexible element interfaces) must account for the three new equations: $(\Re_1 = 0, \Re_2 = 0, \Re_2 = 0)$, to guarantee solution fidelity.

A possible development path for the methods of solid mechanics is sketched in the figure next to the pie-chart. The compatibility barrier prevented the progress of the force method for indeterminate problems. It bifurcated into the redundant force method, which has met with its demise. The stiffness method is compatibility noncompliant especially at the numerous boundaries in a finite element model. The direct force determination method titled Integrated Force Method has been specialized to obtain the displacement method, the mixed method and total formulation. The displacement method cannot be transformed to obtain IFM. In summary, the theory of solid mechanics was not quite complete with respect to the compatibility condition. The benefits that accrue from a use of the compatibility conditions have been shown in:

- Finite element analysis via primal and dual Integrated Force Methods.
- In elasticity through the Completed Beltrami Michell's Formulation.
- In design optimization via IFM.
- The analysis formulations are expected to converge to:
  (a) Method of forces, or the Integrated Force Method: (primal method).
  (b) Displacement method, or Dual Integrated Force Method: (dual method).

Publications

Only ten journal publications are listed. Technical reports and conference presentations are not included.

1. D.A. Hopkins, R.M. Coroneos, G.R. Halford, and S.N. Patnaik. "Fidelity of the Integrated Force Method Solution," Letter to Editor, *Int. Jnl. Numerical Methods in Engrg.*, vol. 55, (2002), pp.(1367–1371).
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4. S.N. Patnaik, J.D. Guptill, D.A. Hopkins, and T.M. Lavelle, "Optimization for Aircraft Engines with Regression and Neural Network Analysis Approximators," *AIAA J. Propulsion*, vol. 35, (1998), pp. (839–850).
5. S.N. Patnaik, and D.A. Hopkins, "General-purpose optimization method for multidisciplinary design application," *Int. Jnl. of Advances in Engrg. Software*, vol. 31, (2000), pp. (57–63).
6. S.N. Patnaik, R.M. Coroneos, and D.A. Hopkins, "Substructuring for Structural Optimization in a Parallel Processing Environment," *Journal of Computer-Aided Civil and Infrastructure Engrg*, vol. 15, (2000), pp. (209–206).
7. S.N. Patnaik, J.D. Guptill, D.A. Hopkins, and T.M. Lavelle, "Neural Network and Regression Approximations in High Speed Civil Transport Aircraft Design Optimization," *AIAA J. Aircraft*, vol. 35, (1998), pp. (839–850).
8. S.N. Patnaik, and D.A. Hopkins, "Recent Advances in the Method of Forces: Integrated Force Method of Structural Analysis," *Int. Jnl. of Advances in Engrg. Software*, vol. 29, (1998), pp. (468–475).
9. S.N. Patnaik, and D.A. Hopkins, "Construction of Finite Elements for the Integrated Force Method," NASA Tech Brief, vol. 21, (1997), pp. (70–73).
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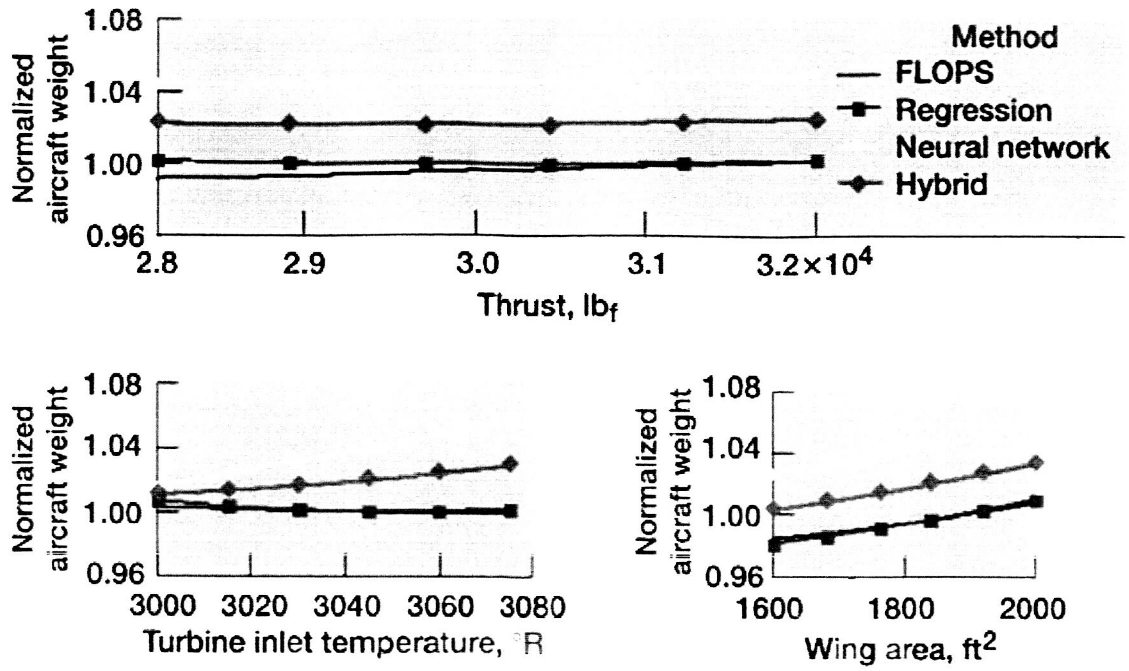
CHAPTER 3
Five Highlights

Highlight-1
Neural Network and Regression Methods Demonstrated in the Design Optimization of a Subsonic Aircraft

The neural network and regression methods of NASA Glenn Research Center's COMETBOARDS design optimization testbed were used to generate approximate analysis and design models for a subsonic aircraft operating at Mach 0.85 cruise speed. The analytical model is defined by nine design variables: wing aspect ratio, engine thrust, wing area, sweep angle, chord-thickness ratio, turbine temperature, pressure ratio, bypass ratio, fan pressure; and eight response parameters: weight, landing velocity, takeoff and landing field lengths, approach thrust, overall efficiency, and compressor pressure and temperature. The variables were adjusted to optimally balance the engines to the airframe. The solution strategy included a sensitivity model and the soft analysis model. Researchers generated the sensitivity model by training the approximators to predict an optimum design. The trained neural network predicted all response variables, within 5-percent error. This was reduced to 1 percent by the regression method.

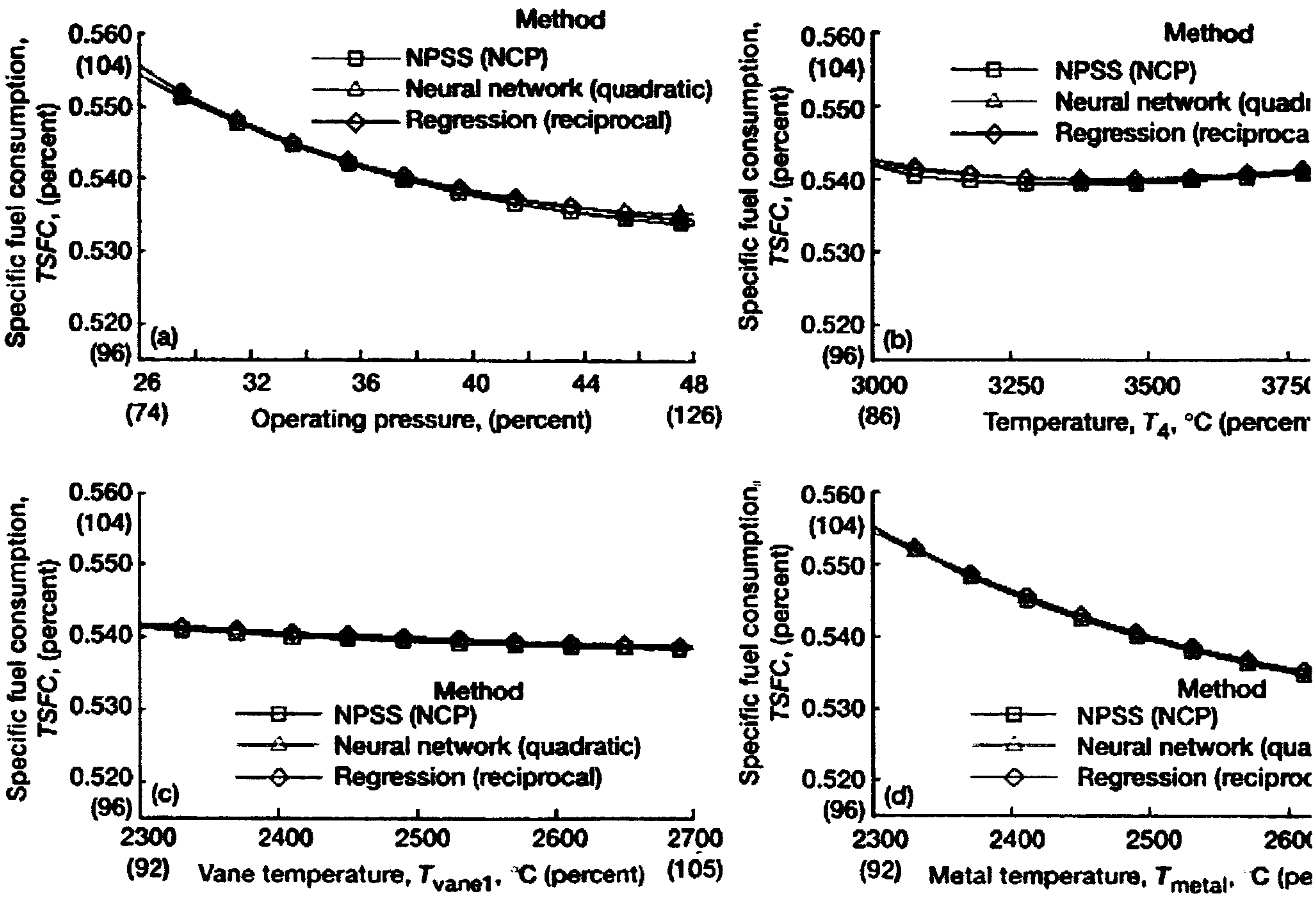
The soft analysis model was developed to replace aircraft analysis as the reanalyzer in design optimization. Soft models have been generated for a neural network method, a regression method, and a hybrid method obtained by combining the approximators. The performance of the models is graphed for aircraft weight versus thrust as well as for wing area and turbine temperature. The regression method followed the analytical solution with little error. The neural network exhibited 5-percent maximum error over all parameters. Performance of the hybrid method was intermediate in comparison to the individual approximators. Error in the response variable is smaller than that shown in the figure because of a distortion scale factor. The overall performance of the approximators was considered to be satisfactory because aircraft analysis with NASA Langley Research Center's FLOPS (Flight Optimization System) code is a synthesis of diverse disciplines: weight estimation, aerodynamic analysis, engine cycle analysis, propulsion data interpolation, mission performance, airfield length for landing and takeoff, noise footprint, and others.



Performance of neural network and regression methods for a subsonic aircraft. Top: Normalized aircraft weight versus thrust. Bottom left: Weight versus turbine inlet temperature. Bottom right: Weight versus wing area.

Highlight-2
Neural Network and Regression Soft Model Extended for PX-300 Aircraft Engine

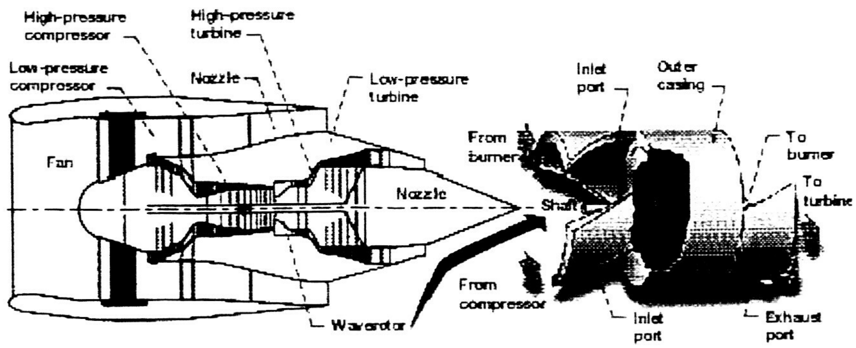
In fiscal year 2001, the neural network and regression capabilities of NASA Glenn Research Center's COMETBOARDS design optimization testbed were extended to generate approximate models for the PAX-300 aircraft engine. The analytical model of the engine is defined through nine variables: the fan efficiency factor, the low pressure of the compressor, the high pressure of the compressor, the high pressure of the turbine, the low pressure of the turbine, the operating pressure, and three critical temperatures (T4, Tvane, and Tmetal). Numerical Propulsion System Simulation (NPSS) calculations of the specific fuel consumption (TSFC), as a function of the variables can become time consuming, and numerical instabilities can occur during these design calculations. "Soft" models can alleviate both deficiencies. These approximate models are generated from a set of high-fidelity input-output pairs obtained from the NPSS code and a design of the experiment strategy. A neural network and a regression model with 45 weight factors were trained for the input-output pairs. Then, the trained models were validated through a comparison with the original NPSS code. Comparisons of TSFC versus the operating pressure and of TSFC versus the three temperatures (T4, Tvane, and Tmetal) are depicted in the figures. The overall performance was satisfactory for both the regression and the neural network model. The regression model required fewer calculations than the neural network model, and it produced marginally superior results. Training the approximate methods is time consuming. Once trained, the approximate methods generated the solution with only a trivial computational effort, reducing the solution time from hours to less than a minute.



PAX-300 engine performance results compared with the specific fuel consumption (TSFC) calculated by soft models. (a) Operating pressure versus TSFC. (b) T4 versus TSFC. (c) Tvane versus TSFC. (d) Tmetal versus TSFC.

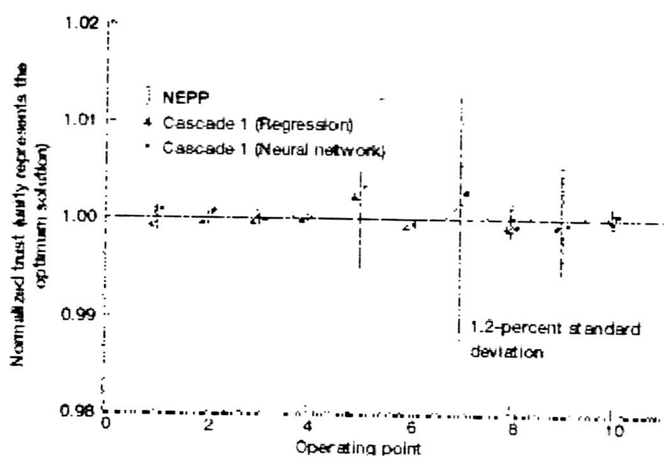
Highlight-3
Engine with Regression and Neural Network Approximators Designed

At the NASA Glenn Research Center, the NASA engine performance program (NEPP, Ref. 1) and the design optimization testbed COMETBOARDS (Ref. 2) with regression and neural network analysis-approximators have been coupled to obtain a preliminary engine design methodology. The solution to a high-bypass-ratio subsonic waverotor-topped turbofan engine, which is shown in the preceding figure, was obtained by the simulation depicted in the following figure. This engine is made of 16 components mounted on two shafts with 21 flow stations. The engine is designed for a flight envelope with 47 operating points. The design optimization utilized both neural network and regression approximations, along with the cascade strategy (Ref. 3). The cascade used three algorithms in sequence: the method of feasible directions, the



Subsonic waverotor-topped gas turbine engine.

sequence of unconstrained minimizations technique, and sequential quadratic programming. The normalized optimum thrusts obtained by the three methods are shown in the following figure: the cascade algorithm with regression approximation is represented by a triangle, a circle is shown for the neural network solution, and a solid line indicates original NEPP results. The solutions obtained from both approximate methods lie within one standard deviation of the benchmark solution for each operating point. The simulation improved the maximum thrust by 5 percent. The performance of the linear regression and neural network methods as alternate engine analyzers was found to be satisfactory for the analysis and operation optimization of air-breathing propulsion engines (Ref. 4).



Optimum solution for the waverotor-topped subsonic engine.

References

1. Klann, John L.; and Snyder, Christopher A.: NEPP Programmers Manual. NASA TM-106575, 1994.

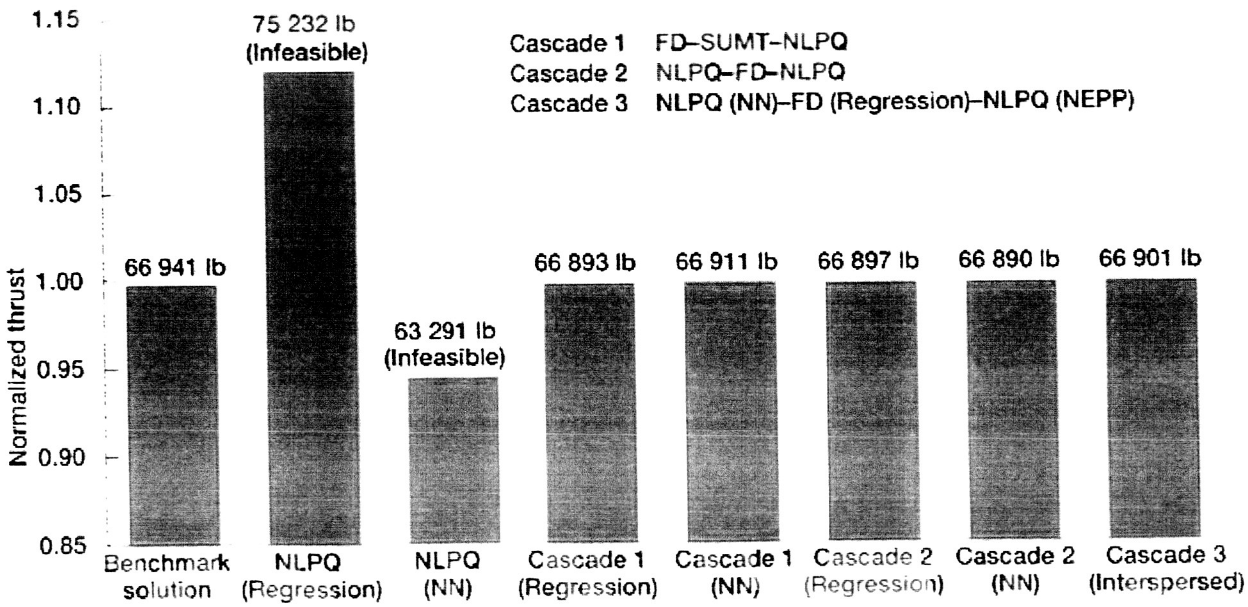
2. Guptill, James D., et al.: CometBoards Users Manual Release 1.0. NASA TM-4537, 1996. http://gltrs.grc.nasa.gov/

3. Patnaik, Surya N.; Coroneos, R.M.; and Hopkins, D.A.: A Cascade Optimization Strategy for Solution of Difficult Design Problems. Int. J. Numer. Meth. Engrg., vol. 40, no. 12, 1997, pp. 2257–2266.

4. Patnaik, Surya N., et al.: Cascade Optimization for Aircraft Engines With Regression and Neural Network AnalysisApproximators. NASA/TM--2000-209177, 2000. http://gltrs.grc.nasa.gov/

Highlight-4
Cascade Optimization Strategy with Neural network and Regression Approximations Demonstrated on a Preliminary Aircraft Engine Design

A preliminary aircraft engine design methodology is being developed that utilizes a cascade optimization strategy together with neural network and regression approximation methods. The cascade strategy employs different optimization algorithms in a specified sequence. The neural network and regression methods are used to approximate solutions obtained from the NASA Engine Performance Program (NEPP), which implements engine thermodynamic cycle and performance analysis models. The new methodology is proving to be more robust and computationally efficient than the conventional optimization approach of using a single optimization algorithm with direct reanalysis. The methodology has been demonstrated on a preliminary design problem for a novel subsonic turbofan engine concept that incorporates a wave rotor as a cycle-topping device. Computations of maximum thrust were obtained for a specific design point in the engine mission profile. The results (depicted in the figure) show a significant improvement in the maximum thrust obtained using the new methodology in comparison to benchmark solutions obtained using NEPP in a manual design mode.



Optimum thrust for a subsonic wave-rotor-topped engine for the sixth operating point.

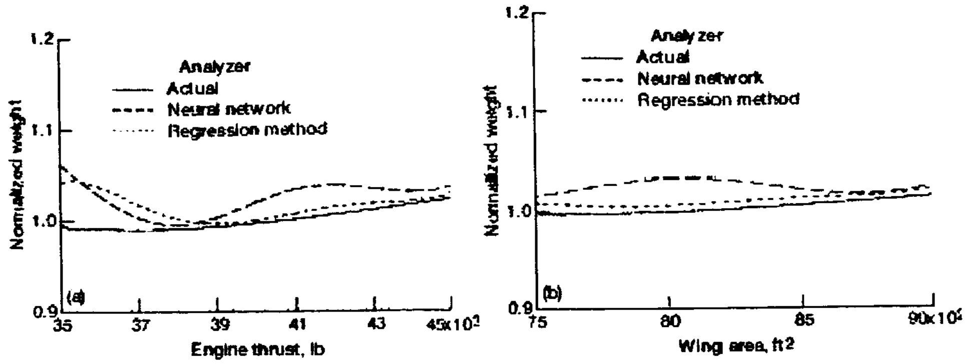
Optimization method	Description
Benchmark solution	Average thrust obtained using 10 different initial designs.
NLPQ (Regression)	Thrust obtained using NLPQ and regression approximation.
NLPQ (NN)	Thrust obtained using the quadratic programming algorithm (NLPQ) and the neural network (NN) approximation.
Cascade 1[a] (Regression)	Thrust obtained using the Cascade 1 strategy and the regression approximation.
Cascade 1[a] (NN)	Thrust obtained using the Cascade 1 strategy and the neural network approximation.
Cascade 2 (Regression)	Thrust obtained using the Cascade 2 strategy and the regression approximation.
Cascade 2 (NN)	Thrust obtained using the Cascade 2 strategy (NLPQ-FD-NLPQ) and the neural network approximation.
Cascade 3 (Interspersed)	Thrust obtained using the interspersed cascade strategy (NLPQ with NN, FD with regression, and NLPQ with the NASA Engine Performance Program (NEPP) reanalysis).

[a]The Cascade 1 strategy uses three algorithms: the Method of Feasible Directions (FD) followed by the Sequential Unconstrained Minimization Technique (SUMT) and the quadratic programming algorithm (NLPQ).

Highlight-5
Neural Network and Regression Approximations Used in Aircraft Design

NASA Lewis Research Center's CometBoards Test Bed was used to create regression and neural network models for a High-Speed Civil Transport (HSCT) aircraft. Both approximation models that replaced the actual analysis tool predicted the aircraft response in a trivial computational effort. The models allow engineers to quickly study the effects of design variables on constraint and objective values for a given aircraft configuration. For example, an engineer can change the engine size by 1000 lb of thrust and quickly see how this change affects all the output values without rerunning the entire simulation. In addition, an engineer can change a constraint and use the approximation models to quickly reoptimize the configuration. Generating the neural network and the regression models is a time-consuming process, but this exercise has to be carried out only once. Furthermore, an automated process can reduce calculations substantially.

One issue that needs to be addressed is the generation of output for a given set of design variables. The analysis tools must be smart enough to respond completely to changes in design values. For example, a change in the overall pressure ratio of an engine will affect several behavior parameters besides an increase in the pressure. For example, the overall efficiency of the compressor will change, the cooling flow temperature will rise, more cooling flow will be required, and the compressor will increase in weight. These problems, however, can be overcome with more capable tools and a little planning.



Comparison of aircraft weight obtained by using approximate methods and actual analysis. (a) As a function of engine thrust. (b) As a function of wing area.

The figure shows how closely the neural network and regression techniques track the output value of aircraft gross weight from the actual analysis. Both approximation techniques have been shown to produce good agreement for all desired response parameters.

CHAPTER 4
Resume of Dr. Surya N. Patnaik, Ph.D.

Three decades-experience in aerospace agencies and educational institutions.

Expertise: Solid Mechanics; Multidisciplinary Design Optimization; Environmental Testing; Management of Aerospace Projects and Personnel; Teaching of Design and Analysis courses.

Education:

Ph.D.: Case Western Reserve University (1972).

Bachelor and Master's degrees from the Indian Institute of Technologies, Bombay and Kanpur, India, respectively.

AIAA Associate fellow (1992); Who'sWho in Science and Engineering, Marquis (1998).

Employment:

India's Space Agency (1972-1988).

Ohio Aerospace Institute through NASA Glenn Research center (1988 to date).

Adjunct Professor: Mechanical Engineering Dept., The University of Akron (1999 to date).

Experience:

Solid Mechanics: Finite element analysis—both Integrated Force Method and Displacement Method.

Multidisciplinary optimization: Structures, Aircraft and Air-breathing propulsion engines.

Sub-problem optimization; Neural Network and Regression approximations as well as Animation.

Smart structures optimization; Probabilistic analysis and optimization.

Testing: Static, dynamic, acoustic and thermal vacuum tests at subsystem and system level for space vehicles.

Management of space vehicle projects: Satellites, Rockets and Earth station.

Teaching Experience:

Finite element analysis and design optimization at Indian Institute of Science, India; University of Arizona, Tucson, Arizona; and University of Moncton, N.B., Canada.

Continuum Mechanics, Theory of Machines and Design Optimization at the University of Akron, Ohio.

Students: Currently I have two full time graduate students. Two MS students graduated in 2003. Their thesis topics: Non-linear plate analysis using force and displacement methods; Probabilistic analysis and optimization; Design of smart beams and plates; Fully utilized method for stress and displacement constraints for uniform and variable depth flexural members.

Managerial Experience:

Chief, Optimization Group in India's Space Agency. Manager for geosynchronous INSAT satellites and earth station projects. Responsible for design, fabrication and qualification testing of INSAT satellites built at Ford Aerospace, CA and launched by Delta, Shuttle and Ariene vehicles. Analysis and design of rocket components.

Current Research Activities: Multidisciplinary optimization and solid mechanics.

Multidisciplinary optimization:
Addressed multidisciplinary design problems as well as the traditional fully utilized design method. The accumulated activity has produced the design optimization testbed titled, 'CometBoards'. It includes:

- Subproblem and substructure strategies to solve large and multidisciplinary problems: Structures, Aircraft, and Aircraft engines.
- Multiple algorithm cascade strategy to assist convergence.
- Design variable and constraint formulations to bypass singularities.
- Neural networks, regression methods and animations to reduce calculations.
- Accounting for scatter in load and material parameters through probabilistic optimization.

  Controlling displacement through the use of smart material. An academic version of CometBoards design optimization tool with a user's manual is available at the website: www.patnaik.mech.uakron.edu. The code has multiple optimization algorithms, stiffness method and Integrated Force Method analyzers. A variety of academic problems can be modeled and solved to learn:
- The fully utilized design concept, optimality criteria and optimization methods can be compared.
- Design improvement from an optimization method, and the price to be paid, can be quantified.
- The merits and limitations of different nonlinear programming algorithms can be examined.
- The suitability of different analysis methods as re-analysis tools in design can be studied.
- The role of sensitivity and consequence of constraint redundancy can be explained.
- The benefit of scaling and design variable and constraint formulations can be shown.

Solid Mechanics:
The strain-formulation that makes solid mechanics a research discipline, remained incomplete since St. Venant's field formulation, ca 1860. We have completed the formulation on the boundary. The boundary compatibility conditions (BCC), unlike the field counterpart, cannot be trivialized. All methods, including the Navier's method have to be adjusted for the compliance of BCC. Our research on strain-formulation has lead to the Integrated Force Method (IFM) for finite element analysis and the Completed Beltrami Michell's Formulation (CBMF) in elasticity. Solution fidelity can be assured via IFM and CBMF because the methods bestow equal emphasis on stress and strain formulations. Benefits from its use have been illustrated in elasticity, in finite element analysis, and in design optimization. A computer code titled 'IFM/Analyzers' that includes both the primal and dual IFM as well as the popular stiffness method has been developed. IFM exhibits potential to complement the stiffness method, balancing the current monopoly biased in favor of the derived method, thus it can spawn the next generation finite element formulation.

The new methods are candidates for 'Curriculum improvement programme' in engineering at graduate and undergraduate levels. We are attempting to modify the undergraduate course "Strength of materials". The textbook has been published.

Strength of Materials: A New Unified Theory for the 21st Century.
Surya Patnaik and Dale Hopkins, Elsevier, October 2003, 750 Pages.

The solution manual for the book has been completed and it is in typing.

The IFM/Analyzer has been reduced to obtain a modest code; entitled IFM-UE: IFM for university (undergraduate) education in engineering. It is available in the website: and it is to accompany the textbook.

Website: http://www.patnaik-ue.org/ifm/

Biography from Marquis Who'sWho in Science and Engineering. (4th Edition, 1998)

PATNAIK, SURYA NARAYAN, research engineer; b. Puri, India, July 22, 1945; came to U.S., 1968; s. Purna Chandra and Kamalini (Dei) P.; m. Padmini Manekarnika, Dec. 4, 1973; children: Mahima, Manish. Btech with honors, Indian Inst. Tech., Bombay, 1966; Mtech, Indian Inst. Tech., Kanpur, 1968; PhD, Case Western Res. U., 1972. Chief Optimization group Vikram Sarabhai Space ctr., Trivandrum, 1983-86; head satellite evaluation group INSAT & Ford Aerospace, Bangalore, India, 1977-82, Palo Alto, Calif., assoc. prof. U. Ariz., Tucson, 1982-83 head IFM Cell Indian Satellite ctr., Bangalore, 1986-1988; NRC rsch. Assoc, NASA Lewis Resc. Ctr., Cleve., 1988-90; sr. rsch. engr. Ohio Aerospace Inst., Cleve., 1990--; mgr. Boundary layer wind tunnel U. Moncton, Can., 1975-77; nat. com. Mem. Software-India, Kanpur, 1986--; cons. Innovative Resch. NASA, NSF, 1990. Contbr. Articles to profl. Jours. Fellow AIAA (assoc. Assn Computational mechanics (session organizer 1995, 97), Internat. Soc. Structures-Multidisciplinary Org., Indian Aero. Soc. Achievements include development of integrated force method, alternate formulation to the current stiffness method, boundary compatibility conditions, a new elasticity equation missed for 100 years, cometboards, an optimization engine for industry. Home: 9107 Highland Dr, Brecksville, OH 44141. Office: Ohio Aerospace Inst 22800 Cedar Point Rd Cleveland OH 44142, (216) 962-3135.